

Non-commutative Dirichlet Forms

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Lecture 1. "General theory of Dirichlet forms on C^* -algebras and von Neumann algebras and KMS-symmetric Markovian semigroups". It is suggested a general knowledge of the notion of C^* -algebras and von Neumann algebras (required or provided to follow the Lectures of Prof. Quanhua Xu) and the main tools of Tomita-Takesaki theory, as those provided by Chapter 2.5 of the book "Operator Algebras and Quantum Statistical Mechanics Vol 1" 2nd edition by O. Bratteli-D.W. Robinson, Springer-Verlag ed.

Lecture 2. "Dirichlet forms and Differential calculus on C^* -algebras". This lecture requires only some general definitions of the representation theory of C^* -algebras and the knowledge of the notion of decomposition of representations. All of that may be provided by Chapter 2.3 of the above cited book.

Lecture 3. "Non commutative Potential Theory and dynamical characterizations of Riemannian manifolds with positive curvature". This lecture requires some tools of potential analysis on manifolds as those involved in the Bochner technique and Hodge's theory in Riemannian manifolds (Clifford algebra and Dirac operator). This can be found in Chapter 7 of the book "Riemannian Geometry" by P. Petersen Graduate Text in Mathematics, 1998 Springer-Verlag ed. as well as in Chapter II-8 of the book "Spin Geometry" by H.B. Lawson-M.-L. Michelson, Princeton Univ. Press 1989.

Lecture 4. "Construction of dissipative dynamics statistical systems". It is suggested only the knowledge of the notion of quasi-free-state on CAR and CCR algebras as contained in Chapter 5.2 of the book "Operator Algebras and Quantum Statistical Mechanics Vol. 2" 2nd edition by O. Bratteli-D.W. Robinson, Springer-Verlag ed.

Lecture 5. "Dirichlet forms and Noncommutative Geometry". As far as possible the lecture will provide the basic properties and examples of two main tools of NCG, namely Fredholm modules and Dirac operator as well their connection with Dirichlet forms. It is suggested some contact with the introductory

parts of Chapter IV "Quantized Calculus" and Chapter VI "The matrix aspect on NCG" of the book "Noncommutative Geometry" by A. Connes, Academic Press 1994.