

Quantum Probabilistic Methods in Operator Space Theory

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This course consists of two parts. In the first, we will give a brief introduction to operator spaces. We will discuss in detail some fundamental examples of operator spaces such as row, column spaces and Pisier's Hilbertian operator space OH . The completely bounded maps on (or between) these spaces will be described. Pisier's theory of vector-valued noncommutative L_p -spaces (mainly Schatten classes) will be also sketched.

In the second part we will try to illustrate the usefulness of quantum probabilistic methods in operator space theory and to show interactions between these two theories. The topics of this part will be mainly concentrated on various noncommutative Khintchine type inequalities. These include Lust-Piquard/Pisier's Khintchine inequalities for Rademacher sequences as well as several free Khintchine inequalities. As applications, we will present, in particular, Junge's embedding of OH into the predual of a von Neumann algebra via a free Khintchine inequality.

References

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