Bioinformatics
(Graph Products and Phenotypes)

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“The Origin of Species” by Charles Darwin (1859) - About the variability of phenotypic characters.
Ernst Haeckel, 1868

Taken from the Book “Kunstformen der Natur” by Ernst Haeckel (1904).
Judging from the number of cervical vertebrae the length of the neck of a giraffe and a human is equal.
What is a character (Merkmal)?

Darwin’s finches

Seeds  Insects

Characters can vary
What is a character (Merkmal)?

Darwin’s finches

**Seeds**  **Insects**

Characters can vary

Mendel’s peas

Characters are independent and can be freely combined
What is a character (Merkmal)?

Characters can vary

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Characters are independent and can be freely combined

Genes and their variants determine the characteristics of a character
What is a character (Merkmal)?

Phenotype = Set of characters with its respective characteristics which can vary independently from each other

Is there a way to determine the characters the phenotypes consists of?
The idea behind

Accessibility of genotypes based on "variational operators" $u \in \mathcal{U}$:

$$X \curlywedge \mathcal{U} \ y$$

$U$-neighborhood of genotypes $y \in X$:

$$N_{\mathcal{U}}(y) := \{x \in X \mid x \curlywedge \mathcal{U} \ y\}$$

Genotype-Phenotype-Map is mapping into phenotype space $\mathbb{P}$

$$f : X \rightarrow \mathbb{P}$$

Accessibility of phenotypes $\alpha, \beta \in \mathbb{P}$:

$$\alpha \curlywedge_{\rho} \beta \iff \frac{|f^{-1}(\alpha) \cap N_{\mathcal{U}}(f^{-1}(\beta))|}{|N_{\mathcal{U}}(f^{-1}(\beta))|} > \rho$$
Genotype-Space
Genotype-Space

Products and Characters
Products and Characters

Genotype-Space

Phenotype-Space

[Diagram showing DNA structures and colored shapes]
Products and Characters

Genotype-Space

Phenotype-Space
Characters and Product Graphs

**Theorem**

*Characters can vary independently (and are thus "real" characters) ⇔ (Local) prime factors of phenotype space*

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Another, but wrong example: