Analysis of epileptic EEG using permutation entropy

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Electroencephalogram (EEG)

Medical explanation

Recording of the voltage fluctuations at the head surface to measure the electrical activity of the brain.

In dependence on age and health, we know some typical rhythms in the EEG, which differ in their frequency:

- $\delta$: $< 4\text{Hz}$
- $\theta$: $4 - 7\text{Hz}$
- $\alpha$: $8 - 13\text{Hz}$
- $\beta$: $14 - 30\text{Hz}$
- $\gamma$: $> 30\text{Hz}$
Arrangement of the electrodes

The examples will each contain the recordings of 19 electrodes and an ECG channel.

Placement of the electrodes according to the 10-20-System:
Epilepsy

Medical explanation

Epilepsy is a chronic disease of the central nervous system. It is characterized by recurrent epileptic seizures, in the form of spasms or involuntary movements as well as semi-consciousness.

- Such attacks are the result of a paroxysmal, abnormally synchronous discharge of neurons in the brain.
- The symptomatology depends on the location and size of the affected area.
Epileptiform potentials (ETP)

It is known that there are some typical waveforms, which occur frequently in the EEG of people who suffer from epilepsy. But there are no quantitative parameters such as amplitude or duration, which allow a reliable differentiation between ETP and other abnormalities.

Helpful criteria for the classification of an ETP

- An ETP contains a sharp wave, which interrupts the basal activity.
- The sharp wave is usually followed by a slow after-variation.
- The electric field distribution can be described as logically.

Some types of epileptic disorders show no ETP in normal EEG. But EEG with ETP does not necessarily prove the presence of epilepsy. (genetic disposition)
Interictal ETP

The following potentials are typical for non-ictal, epileptic EEG:

- Spikes (< 80ms)
- Spike-waves (about 3 Hz, often on the Frontal Lobe, easily triggered by hyperventilation)
- Polyspikes (3-5 seconds in length, mostly in sleep)
- Polyspike-waves
- Sharp-waves (> 80ms mono-, bi or triphasic)
- Sharp-slow-waves (Spike-waves < 3Hz)
Example 1.1

Benign epilepsy (Rolando), awake EEG, right temporal focus
Example 1.2

Same child, Polyspikes due to the slowdown in activity (tired)
Example 1.3

Same child, multiple focus activation in sleep

Analysis of epileptic EEG using permutation entropy
Example 2.1

Bifocal epilepsy, awake EEG, average reference
Example 2.2

Same EEG after Laplace-reference: asymmetric foci, earlier response in the right hemisphere but greater amplitude on the left side.
Example 3.1

Juvenile absence epilepsy (pyknoleptic, primary generalized)

General response, but no absence
Example 3.2

Same EEG, a few seconds later: slight absence
Permutation entropy (PE)


We consider a time series \( \{x_t\}_{t=1}^T \). For given order \( d \) and delay \( \tau \) the delay vector \( \nu_{d,\tau} \) for \( t \) is defined as:

\[
\nu_{d,\tau}(t) = (x_t, x_{t+\tau}, x_{t+2\tau}, \ldots, x_{t+d\tau}), \; t = 1 \ldots T - d\tau
\]

We neglect equal values \( x_{t^*} = x_t \) for \( t^* \neq t \) within the same delay vector. Then we can assign an ordinal number to each value and consider the arrangement of these numbers. There are \( (d + 1)! \) different possible permutations \( \pi \) and for each \( \pi \in \Pi \) we can estimate the relative frequency in the whole time series considering all delay vectors:

\[
p(\pi) = \frac{\#\{t|\nu_{d,\tau}(t) \text{ has type } \pi\}}{T - d\tau}
\]
Definition

The permutation entropy $H_{d,\tau}$ of order $d \geq 1$ and delay $\tau \geq 1$ is given by:

$$H_{d,\tau} = -\sum_{\pi \in \Pi} p(\pi) \log p(\pi)$$

It is easy to see that $0 \leq H_{d,\tau} \leq \log(d + 1)!$

So the entropy can be normalized on $[0, 1] : h_{d,\tau} = \frac{H_{d,\tau}}{\log(d+1)!}$

In the following there will be used the normalized permutation entropy with $d = 3$ and $\tau = 1$. 
Dealing with identical potentials

We have assumed that there are no identical values within the same delay vector. This is justified if the values $x_t$ have a continuous distribution, so that equal values are really rare.

But the present EEG data are measurements of slightest cortical potential changes that need to be strengthened technically and cannot be measured exactly.

Therefore, equal values occur frequently.
Possible solutions

- Add small random numbers, e.g. from the uniform distribution on $[0, 0.001]$
- Simply ignore delay vectors with identical values
- Assign them all to one special permutation (not recommended)
- Define the first of two equal values as the greater one
Choice of reference

There are lots of different types of reference that could be used:

- Average reference
- Nearest neighbour reference
- Central reference
- Horizontal or vertical reference
- Laplace reference
- Reference to the ears
- ...

![Laplace reference, Average reference, Central reference]
How does the kind of reference influence the permutation entropy?
Effects of filters

The permutation entropy of filtered data is lower (of course)
First thesis

The permutation entropy of sections with ETP ist lower than elsewhere. Whether the EEG was previously filtered or not, has no influence, besides the magnitude.
If there are no ETP, the PE is greater in all channels. In the case of a generalized epilepsy the differences in entropy don’t have the same size in every channel.

In case of a focal epilepsy, the differences of entropy are visible everywhere, but greater in the aktive epileptic channels.
Outlook

- Vigilance and age effects:
  - Change of entropy in different sleep stages
  - Effects of sleep spindles or K-complexes
  - Development of $\alpha$-waves

- The best general delay is 1. But are there best delays for special kinds of epilepsy?

- Differences between ictal and interictal PE

- Are effects of epilepsy on the intellect measurable by PE?

- Can PE help if there are changes in epileptic seizures but no visible changes in EEG?
Thank you for listening!