Abstracts for the conference FGS 4

Monday, September, 8 - Friday, September,12 2008

All abstracts of invited and contributed talks in alphabetical order.

Visible parts of fractal percolation

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Joint work with Esa Järvenpää, Maarit Järvenpää and Pablo Shmerkin.

The visible part of a compact set $F \subset \mathbb{R}^n$ from an affine subspace K of $\mathbb{R}^n$ consists of those points $x \in F$ for which $[x, P_K(x)] \cap F = x$, where $P_K(x)$ is the orthogonal projection of x to the affine subspace K and $[x, P_K(x)]$ is the line segment joining x to $P_K(x)$.

Fractal percolation (in the plane) is constructed by dividing the unit square into M^2 subsquares of equal size and choosing every square independently of each other with probability p to be included in F_1. In the next stage we divide each chosen square again into M^2 subsquares and choose each of them independently with probability p to be included in F_2. Continuing this, we get a random fractal $F = \cap_{i=1}^{\infty} F_i$.

It is an open question if the dimension of the typical visible part of a compact subset of $\mathbb{R}^n$ is $n-1$ if the dimension of the set is larger than $n-1$. For fractal percolation we prove that this is true with probability one when probability parameter p is larger than or equal to percolation threshold p_c i.e. when there is a connected component from the left side to the right side of the unit square with positive probability. We also prove that with probability one the visible part from the x-axis has dimension less than or equal to one for all p.

On the local time of Linear Fractional Stable Sheet

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Joint work with Francois Roueff (Telecom Paris ParisTech), Yimin Xiao (Michigan State University).

Linear Fractional Stable Sheet (LFSS) is an anisotropic multivariate stable extension of Fractional Brownian Motion. Heuristically speaking, it can be viewed as a tensor product of FBM's. However, there exists an essential difference between these two objets: LFSS does not satisfy the property of local nondeterminism (heuristically speaking this property means that the increments are locally approximatively independent). The main goal of our talk is to present some strategies allowing to overcome these difficulty and to prove the existence of the local time of LFSS and its joint continuity. These strategies mainly rely on a nice decomposition of LFSS that we will introduce.

The diffraction measure of the Thue-Morse chain

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Joint work with Uwe Grimm.

The classical Thue-Morse chain is an important example of symbolic dynamics. When realised as an element of $\{-1, 1\}^{\mathbb{Z}}$, it has a purely singular continuous diffraction measure. This was first proved by Kakutani in 1972. Many numerical and recursive approaches to the actual measure can be found in the literature, but an exact calculation seems to be missing.

In this joint work with Uwe Grimm, an exact formula is derived for the distribution function of the diffraction measure. Since the measure is $\mathbb{Z}$-periodic, the entire analysis takes place on the unit interval. Similarities and differences to the Cantor measure are briefly discussed.

Transformations between fractals

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This talk provides an overview of some theory and applications of transformations between attractors of hyperbolic and fibre-limit iterated function systems. The goal is to introduce the main elements of a new methodology for the practical construction of a rich class of fractal transformations between subsets of $\mathbb{R}^n$. Fractal transformations generalise fractal interpolation: they may change fractal dimensions while preserving topological structure, and they have a number of interesting applications.)

First, conditions and a theoretical construction are provided, yielding natural continuous transformations from one IFS attractor into another. Two IFS attractors are equivalent if their code space structures are the same: but how does one ensure this, for an IFS of affine or projective transformations in $\mathbb{R}^n$? A partial answer is provided by several examples of parameterized families of IFSs, with constant equivalence class stuctures and associated continuous dynamical systems. Second, we are led to the question: when does an IFS, that promises to admit a particular code space structure, actually possess a unique attractor? A partial but useful answer is provided by an extension to some work of Kameyana on topologically self-similar fractals; in the case of affine maps we exploit Minkowski distance functions and in the case of projective transformations we make use of Hilbert geometries.

Third, assured of convergence, we show how a modified chaos game can be applied to compute the graphs of fractal transformations. In particular, in the cases of affine and projective IFSs in two dimensions, we are able to compute high resolution pictures that illustrate homeomorphisms between attractors of IFSs. (These pictures are very different from popular fractal pictures, and appear to me to be extraodinarily beautiful.)

Fourth, what can be said of the smoothness and other properties of the fractal transformations? A model example, associated with v-variable fractal interpolation, is provided in which the releavant Hölder exponents can be computed exactly.

Parts of this talk refer to joint work and/or discussions with Ross Atkins, Uta Freiberg and David Wilson (conditions under which affine and projective IFSs are fibre-limit), and Peter Massopust (Hölder exponents of some families of fractal homeomorphisms).

The singularity spectrum for the inverse of Gibbs measures on cookie-cutter sets

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A Gibbs measure on a cookie-cutter set satisfies the multifractal formalism. We show that the same holds for the discrete measures obtained as its inverse.

Self regulating multifractional process and applications

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Joint work with Jacques Lévy Véhel.

The idea is to build a process Z whose local regularity α_Z is a function of the value of the process, i.e such as : $\alpha_Z(t) = g(Z(t))$ for all t, almost surely. This model is motivated by the observation of natural phenomenon, for which we observe that the regularity depends on the values of the phenomenon. For example, a mountain, whose altitude is rather high, is less regular than a valley, whose altitude is lower. We also noticed that the heart rate tends to be more irregular when the heart beats slower, usually during night. The construction of such a process uses a Banach fixed point theorem, and is based on a field of fractional Brownian motions. This provides both the existence of the process, and a method of synthesis.

On Brownian Flights and their time-schedule

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We consider domains with irregular (fractal) boundaries. We choose a point in the domain at random at distance ϵ from the boundary and start Brownian motion which we stop when it hits the boundary. We study statistics of the length and duration of this diffusion.

Measure-valued diffusions, self-similarity and flickering random measures

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Measure-valued diffusions are stochastic processes that arise as limits of empirical measures of interacting stochastic particle systems. Important ingredients of such systems are the underlying motion (resp. mutation) of particles and their reproduction mechanism.

While such models have interesting applications in mathematical genetics and population biology, they also reveal a rich mathematical structure. Sophisticated methods have been introduced to construct and explore these processes, e.g. Bertoin and Le Gall's flow of bridges and Donnelly and Kurtz' modified lookdown construction.

In this talk we show how the modified lookdown construction can be used to reveal interesting structural- and path-properties of measure-valued diffusions, in particular in the case when the underlying motion and reproduction mechanisms satisfy certain self-similarity properties. As a spin-off, we arrive at the notion of a 'flickering random measure'.

Directed porosity on conformal iterated function systems and singular integrals

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We discuss directed porosity in connection with conformal iterated function systems (CIFS) and with singular integrals. We show that limit sets of finite CIFS are porous in a stronger sense than already known. Furthermore we use directed porosity to establish that truncated singular integral operators, with respect to general Radon measures μ and kernels K, converge weakly in some dense subspaces of $L^2(\mu)$ when the support of μ belongs to a broad family of sets including finite CIFS's limit sets.

Problems in geometric measure theory

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Joint work with G. Alberti and D. Preiss.

Our main goal in this talk is to describe non-differentiability sets of Lipschitz functions on $\mathbb{R}^n$, and to understand the phenomena that occur because of behaviour of Lipschitz functions around the points of null sets.

We will show how our results and techniques can be used to solve various other problems in analysis, and we will discuss some combinatorial connections and the main problems that these results left open.

Algebraic difference of random Cantor sets II

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The study of the algebraic difference

$$F_2 - F_1 = \{y - x : x \in F_1, y \in F_2\}$$

of two dynamically defined Cantor sets $F_1, F_2 \subset \mathbb{R}$ was motivated by the reserch of Palis and Takens in regards with the unfolding of homoclinic tangency in some one-parameter families of surface diffeomorphisms.

Palis conjectured that if

$$\dim_{\mathrm{H}} F_1 + \dim_{\mathrm{H}} F_2 > 1$$

then *generically* it should be true that

$$F_2 - F_1 \text{ contains an interval.}$$

For generic dynamically generated *non-linear* Cantor sets this was proved in 2001 by de Moreira and Yoccoz. The problem is open for generic linear Cantor sets. In this talk I will speak about related results for random Cantor sets.

On an imbedding theorem concerning Lipschitz and Lorentz spaces on h-sets

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Let $F \subset \mathbb{R}^n$ be endowed with a measure μ satisfying for some $d, C_1 > 0$

$$\mu(\{y \in F : |x-y| < r\}) \geq C_1 r^d \quad \text{for all } 0 < r \leq 1 \text{ and } x \in F.$$

For example F may be a d-set, or a h-set with $h(r) \geq r^d$. We show that there is an imbedding

$$\mathrm{Lip}(\alpha, p, q, F) \subset L_{p^*,q}$$

for $\alpha \in (0,1]$, $0 < p \leq q < \infty$ and $p^* = pd/(d - \alpha p)$. This extends the result in Proposition 6 in [A. Jonsson and H. Wallin, *Function spaces on subsets of* $\mathbf{R}^n$, *Math. Rep.*, 2(1):xiv+221, 1984; page 216]. We have also the following imbedding for any $\alpha > 0$

$$\mathrm{Lip}_0(\alpha, p, q, F) \subset L_{p^*,q},$$

where

$$\mathrm{Lip}_0(\alpha, p, q, F) = \{f : \{\tilde{f}^{(j)}\} \in \mathrm{Lip}(\alpha, p, q, F)\}$$

and $\tilde{f}^{(0)} = f$, $\tilde{f}^{(j)} = 0$ for multiindices j with length $|j| > 0$.

Multistable Processes

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Joint work with Jacques Levy Vehel and Ronan Le Guével.

We describe a general method for constructing stochastic processes with prescribed local form, encompassing examples such as variable amplitude multifractional processes. In particular we use Poisson sums to construct multistable processes, that is processes that are locally alpha(t)-stable but where the stability index alpha(t) varies with time t.

Spectral dimension on V-variable fractals

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Joint work with John Hutchinson.

The concept of V-variable fractals (developed by Barnsley, Hutchinson & Stenflo) allows describing new families of random fractals, which are intermediate between the notions of deterministic and of random fractals including random recursive as well as homogeneous random fractals. The parameter V describes he degree of "variability" of the realizations.

Brownian motion and Laplacian can be constructed from the associated Dirichlet forms. The properties of these objects are modified by the random environment. We obtain the spectral dimension (i.e. the exponent of the leading term of the eigenvalue counting function of the Laplacian) by applying Kesten-Furstenberg techniques. If time allows, we sketch some results on Hausdorff and walk dimension. Hence, we discuss how the degree of variability influences the geometric, analytic and stochastic behavior of the fractals.

Fractal fundamental domains for lattices in the plane

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The point group of a lattice is the group of linear isometries mapping the lattice to itself. It is easy to see that each lattice possesses a fundamental domain (wrt to translations) which symmetry group is the point group of the lattice: just take

the fundamental parallelotope. It is kind of surprising that there are fundamental domains for certain lattices which possess more symmetry than the point group of the corresponding lattice. For instance, the point group of the square lattice is D_4 (the dihedral group of order 8). But there is a fundamental domain of the square lattice with eightfold symmetry, its symmetry group being D_8. This fundamental domain has a fractal shape. It was discovered by Veit Elser some 10 years ago. In this talk we show that almost all plane lattices have fundamental domains with more symmetry than the point group of the underlying lattice. In the generic case, this fundamental domain will be of fractal shape. We show some properties of these fractals, together with a lot of nice pictures. If there is time, we will conclude with an outlook to higher dimensions.

Dimension functions of Cantor sets

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Joint work with Ursula Molter and Roberto Scotto.

Cabrelli *et al* [3], based in the paper of Besicovitch and Taylor [1], showed that every Cantor set C_a associated to a non-increasing sequence a is dimensional; that is, they constructed a function $h_a \in \mathscr{D}$, the set of dimension functions, for which C_a is an h_a-set, i.e., $0 < \mathcal{H}^{h_a}(C_a) < +\infty$, been $\mathcal{H}^{h_a}$ the h_a-dimensional Hausdorff measure. Moreover, they show that if the sequence a behaves like $n^{-1/s}$, then $h_a \equiv x^s$ and therefore C_a is an s-set. But in other cases the behavior of these functions is not so clear. For example, there exists a sequence a such that C_a is an α-set but $h_a \not\equiv x^\alpha$ [2]. So these functions could be too general in order to give a satisfactory idea about the size of the set. To understand this situation we study the packing premeasure of these sets. We are able to characterize completely when a dimension function is equivalent to a h_a. More precisely, for $g \in \mathscr{D}$ we obtain that

$$g \equiv h_a \iff 0 < \mathcal{H}^g(C_a) \leq P_0^g(C_a) < +\infty,$$

where P_0^g is the g-dimensional packing premeasure. Thus, to have that $x^\alpha \equiv h_a$, it is not only necessary that C_a is an α-set but also that $P_0^\alpha(C_a) < +\infty$.

[1] A. S. Besicovitch and S. J. Taylor. On the complementary intervals of a linear closed set of zero Lebesgue measure. *J. London Math. Soc.*, 29:449–459, 1954.

[2] Carlos Cabrelli, K Hare, and Ursula M. Molter. Some counterexamples for cantor sets. *Unpublished Manuscript.*, Vanderbilt 2002.

[3] Carlos Cabrelli, Franklin Mendivil, Ursula M. Molter, and Ronald Shonkwiler. On the Hausdorff h-measure of Cantor sets. *Pacific J. Math.*, 217(1):45–59, 2004.

Connectivity of Pisot dual tilings

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Joint work with Shigeki Akiyama.

We have studied the connectedness of Pisot dual tilings which play an important role in the study of β-expansion, substitution and symbolic dynamical system. It is shown that each tile generated by a Pisot unit of degree 3 is arcwise connected. This is naturally expected since the digit set consists of consecutive integers. However surprisingly, we found families of disconnected Pisot dual tiles of degree 4 which have infinitely many connented components. Also we give a simple necessary and sufficient condition for the connectedness of the Pisot dual tiles of degree 4. As a byproduct, a complete classification of the β-expansion of 1 for quartic Pisot units is given.

Barnsley's Fern

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Barnsley's Fern is determined by four affine maps, two of which are similitudes, and one of which is nearly degenerate. The remaining one is not conformal, but its eigenvalues are conjugate complex, and therefore have the same modulus. This allows us to introduce its largest Liapunov exponent as a reduction factor playing the same role as the similarity ratio of a similitude. Ignoring the nearly degenerate map, we can apply techniques drawn from the theory of self-similar fractals and define a local Lyapunov dimension, compare it to the spherical dimension and find a lower bound for the Hausdorff dimension of the Fern.

The dichotomy in the heat kernel two-sided estimates

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We characterize the two-sided estimates for the axiomatically defined heat kernels on metric measure spaces, which depend on a certain space/time scaling. The main result is the dichotomy of such estimates: either it is of a sub-Gaussian type (like the heat kernels of diffusions on the fractals spaces), or it has a polynomial tail (like the heat kernels of the symmetric stable processes in the Euclidean spaces).

Kernels of fractal star bodies

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Joint work with M. Moszyńska, D. Pronk ([1]).

We say that a body $A \in \mathbb{R}^n$ is a *fractal star body* (*with respect to some fractal dimension* $\dim_F$) if and only if $\operatorname{bd} A$ is a fractal (with respect to $\dim_F$). We shall discuss the topological dimension of kernels of fractal star bodies.

[1] I. Herburt, M. Moszyńska, D. Pronk, Fractal star bodies, Proc. conf. Convex and Fractal Geometry, Banach Centre Publ. (to appear).

Stochastic partial differential equations with fractal gradient type noise

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Joint work with Martina Zähle (Jena).

In the talk, we present a pathwise finite-dimensional approach to (systems of) parabolic partial differential equations perturbed by certain low order (random) noises of fractional Brownian nature. We use fractional calculus and some Fourier analysis to construct Stieltjes type integrals that are well suited to PDE theory. Solutions to systems and equations are defined in the mild (semigroup) sense. Under natural assumptions their existence and uniqueness follow from a contraction principle. Nonlinearities and products of distributions are formulated in terms of Fourier decompositions.

Lower bound of heat kernel and parabolic Harnack inequality

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The relationship between the lower bound of heat kernel and parabolic Harnack inequality is discussed.

V-variable fractals and their analysis

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Joint work with Michael Barnsley, Örjan Stenflo and Uta Freiberg.

V-variable fractals, for natural numbers V, provide classes of random fractals with nice properties which interpolate between random homogeneous fractals and random recursive fractals. I will discuss some of their dimension and analytic properties.

Fractal tops for one family of iterated function systems

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We consider the one-parameter family of iterated function systems (IFS) on the unit interval: $([0,1], g_0, g_1)$, where $g_0(x) = tx$, $g_1(x) = tx + 1 - t$ are contractions on $[0,1]$ and $t \in (0,1)$ is a parameter. Let us denote by A_t the attractor of this system. An element $\omega \in \prod_0^\infty \{0,1\}$ is called address of $x \in A_t$ if $x = \lim_{j\to\infty} g_{\omega_0} \circ g_{\omega_1} \circ \cdots \circ g_{\omega_j}(0)$. It is easily seen that $x = (1-t)\sum_{j=0}^{\infty} \omega_j t^j$.

For $t \in (\frac{1}{2}, 1)$ almost all points $x \in A_t$ have more than one address. The set of addresses of a point $x \in A_t$ possesses a unique lexicographically largest element. Following Barnsley we call this element the top at t and denote the set of all tops at t by Ω_t.

Given address τ we find a set of $t \in (0,1)$ such that $\tau \in \Omega_t$. Then we describe Ω_t and give the explicit construction of this set in a special case.

Self-affine sets of Kakeya type

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Joint work with P. Shmerkin.

We compute the Minkowski dimension for a family of self-affine sets on $\mathbb{R}^2$. Our result holds for every (rather than generic) set in the class. Moreover, we exhibit explicit open subsets of this class where we allow overlapping, and do not impose any conditions on the norms of the linear maps. The family under consideration was inspired by the theory of Kakeya sets.

Motion In Fractal Brane World Models

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Path and path deviation equations for test particles and spinning charged objects in fractal brane world models are obtained. We examine the transitional case of their corresponding equations in a four-dimensional fractal space-time. We also obtain a fractal version of path and path deviation equations for the above mentioned objects defined in curved Clifford space.

Self-similarity and random walk

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We shall discuss a new development at the crossroads of Analysis, Algebra and Probability related to self-similarity. Amenability (its definition going back to von Neumann) is, from the analytical point of view, the most natural generalization of finiteness or compactness. Namely, amenable groups are those which admit an invariant mean (rather than an invariant probability measure, which is the case for finite or compact groups). Groups acting by automorphisms of a homogeneous rooted tree (self-similar and automata groups, iterated monodromy groups) has recently become the object of an extensive study in the group theory, since even in the simplest situations such groups may have rather unusual properties. We shall describe a new technique for proving amenability of self-similar groups ("Munchhausen trick") developed by the author and based on using the notion of the asymptotic enropy of a random walk. This technique has recently lead to a proof of amenability for a large class of self-similar groups by Bartholdi, Nekrashevych, Virag and the author.

Weyl type spectral asymptotics for the Laplacian on Sierpinski carpets

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In this talk, I will talk about a detailed asymptotic behavior of the eigenvalues of the Laplacian on Sierpinski carpets.

There are many results for construction of Laplacians on self-similar sets, and it is known that there exist good strong local regular Dirichlet forms on a class of finitely ramified self-similar sets (nested fractals, such as the Sierpinski gasket) and also on a class of Sierpinski carpets.

For such a Dirichlet form on a self-similar set, the corresponding non-negative self-adjoint operator ('Laplacian') is shown to have compact resolvent, so its spectrum is written uniquely in the form $\{\lambda_n\}_{n=1}^{\infty}$ of a non-decreasing sequence tending to infinity. The eigenvalue counting function $N(x)$ of this Laplacian is defined as the number of eigenvalues less than or equal to x. We would like to know the asymptotic behavior of $N(x)$ as x tends to infinity.

For finitely ramified self-similar sets, such kinds of results were known already in 90's. Recently for the Laplacian on Sierpinski carpets, B. M. Hambly (Asymptotics for functions associated with heat flow on the Sierpinski carpet, preprint) has proved a similar result on the asymptotic behavior (as $t \downarrow 0$) of the trace

$$T(t) := \sum_{n=1}^{\infty} e^{-\lambda_n t}$$

of the corresponding heat semigroup instead of $N(x)$, using arguments on the diffusion process and the sub-Gaussian estimate of the transition density. All these results mainly concern the principal term of the asymptotic behavior of $N(x)$ (as x tends to infinity) or $T(t)$ (as t tends to 0).

The main result of this talk describes the asymptotic behavior (as t tends to 0) of

$$T(t) - \text{(the principal term)},$$

namely: For the case of the Laplacian on nested fractals or Sierpinski carpets, $T(t)$ – (the principal term) admits an asymptotic behavior similar to the principal term (and the same is partly true for higher order terms).

Applications of geometric measure representations in cases of simplicial or spherical distributions

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The density level sets of the two types of measures under consideration are $l_{n,p}$-spheres with $p = 1$ or $p = 2$, respectively. The intersection-percentage function (i.p.f.) of such a measure reflects the percentages which the level set corresponding to the p-radius r shares for each $r > 0$ with a set to be measured. The geometric measure representation formulae in [1] and [2] are based upon these i.p.f.'s and will be used here for evaluating exact distribution functions for the products of the components of two-dimensional (n=2) simplicial or spherically distributed random vectors.

To this end, the talk starts with studying the two i.p.f.'s. In the next step, the resulting probability integral representations will be considered and several analytical and numerical conclusions will be presented. The integral representation formulae allow modeling distributions having light or heavy tails. Thus, applications deal with different density generation functions, which can be inserted into the geometric representation formulae. To illustrate and compare their effects, the results of a corresponding numerical study will be presented.

In the case of the exponential law, the probability-integral turns out to be the exact solution of the Bessel differential equation. Hence, a geometric representation for the modified Bessel function of the second kind may be derived from the present results.

[1] Henschel,V., Richter,W.-D., Geometric generalization of the exponential law. J. Mult. Anal. 81(2002),189-204; doi:10.1016/jmva.2001.2001.
[2] Richter,W.-D., A geometric approach to the Gaussian law, in "Symposia Gaussiana, Conf. B" (V. Mammitsch and H. Schneeweiß, Eds.) pp. 24-45, de Gruyter, Berlin/New York, 1995.

The measure of the intersection of two copies of a self-similar or self-affine set

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Joint work with Márton Elekes and András Máthé.

Let $K \subset \mathbb{R}^d$ be a self-similar or self-affine set, let μ be a self-similar or self-affine measure on it, and let $\mathcal{G}$ be the group of affine maps, similitudes, isometries or translations of $\mathbb{R}^d$. Under various assumptions (such as separation conditions or we assume that the transformations are small perturbations or that K is a so called Sierpiński sponge) we get results of the following types, which are closely related to each other.

- (*Non-stability*)

  There exists a constant $c < 1$ such that for every $g \in \mathcal{G}$ we have either $\mu\big(K \cap g(K)\big) < c \cdot \mu(K)$ or $K \subset g(K)$.

- (*Measure and topology*)

  For every $g \in \mathcal{G}$ we have $\mu\big(K \cap g(K)\big) > 0 \iff \mathrm{int}_K(K \cap g(K)) \neq \emptyset$ (where int_K is interior relative to K).

- (*Extension*)

  The measure μ has a $\mathcal{G}$-invariant extension to $\mathbb{R}^n$.

Moreover, in many situations we characterize those g's for which $\mu\big(K \cap g(K)\big) > 0$ holds, and we also get results about those g's for which $g(K) \subset K$ or $g(K) \supset K$ holds.

On the fractal structure of ordinal time series analysis

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Joint work with Mathieu Sinn (Universität Lübeck, Germany).

Ordinal time series analysis is a new promising approach to the qualitative investigation of long and complex time series. The idea behind it is to consider the order relation between the values of a time series instead of the values themselves. Roughly speaking, a given time series is transformed into a series of so called ordinal patterns describing the up and down in the original series. Then the distribution of ordinal patterns obtained is the base of the analysis.

Here we focus to the fractal structure of ordinal pattern distributions related to one-dimensional dynamical systems. In particular, we show that these distributions are either fat or very thin in some sense and relate ergodicity of the dynamical system to a special property of the system of ordinal patterns. The obtained results give some new insights into the relationship of the Permutation entropy introduced by Bandt and Pompe and the Kolmogorov-Sinai entropy of a one-dimensional dynamical system.

Hölder irregularity of distribution functions

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Joint work with Bernd O. Stratmann and partly also with T. Jordan and M. Pollicott.

We give applications of the thermodynamic formalism to the theory of distribution functions of Gibbs measures (devil's staircases) supported on fractal limit sets of finitely generated conformal iterated function systems in $\mathbb{R}$. For a large class of these Gibbs states we determine the Hausdorff dimension of the set of points at which the distribution function of these measures is not α-Hölder-differentiable. The obtained results give significant extensions of recent work by Darst, Dekking, Falconer, Li, Morris, and Xiao. We also discuss the case when the support of the singular measure is an interval.

Brownian motion and thermal capacity

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Joint work with Yimin Xiao.

An old folklore problem asks for the hausdorff dimension of the intersection of a brownian image with a set. We give a solution that is related closely to N. Watson's theory of super temperatures (1976).

Our solution hinges on a new notion of fractal dimension that appears to be "very complicated" in dimension one, and yet "very natural" in dimensions two and higher.

The Hausdorff dimension of the double points on the Brownian frontier

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The frontier of a planar Brownian motion is the boundary of the unbounded component of the complement of its range. We find the Hausdorff dimension of the set of double points on the frontier.

Measurable Riemann Geometry on the Sierpinski gasket

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We study the standard Dirichlet form and its energy measure, called the Kusuoka measure, on the Sierpinski gasket as a prototype of “measurable Riemannian geometry”. The shortest path metric on the harmonic Sierpinski gasket is shown to be the geodesic distance associated with the “measurable Riemannian structure”. The Kusuoka measure is shown to have the volume doubling property with respect to the Euclidean distance and also to the geodesic distance. Li-Yau type Gaussian off-diagonal heat kernel estimate is established for the heat kernel associated with the Kusuoka measure.

Multifractal analysis of measures on the boundary of a Galton-Watson tree

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There are two natural measures on the boundary of a Galton-Watson tree, the branching measure and the harmonic measure for simple forward moving random walk. We introduce these measures and investigate their multifractal spectrums.

A classification of Reifenberg-like properties

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Reifenberg's ϵ-approximation property can be stated as follows. Let $\epsilon > 0$, $S \subset \mathbb{R}^m$ be bounded, $\rho_0 > 0$ and $j \leq m$. S is then j-dimensionally ϵ-Reifenberg approximable

if for each $x \in S$ and $\rho \in (0, \rho_0]$ there exists a j-dimensional affine plane $L_{x,\rho}$ such that $d_H(B_\rho(x) \cap S, B_\rho(x) \cap L_{x,\rho}) \leq \epsilon\rho$, where d_H denotes Hausdorff distance.

Reifenberg proved in 1960 that such sets are (essentially) bihomeomorphic to a j-dimensional disk. Since that time, variants of Reifenberg's property have been found to be applicable elsewhere in mathematics, for example in Leon Simon's proof of the rectifiability of a class of minimal surfaces, where stronger properties than bihomeomorphic representation are desired. We present here a systematic list of variants of Reifenberg's property and classify them in terms of whether or not they ensure j-dimensionality, locally $\mathcal{H}^j$-dimensional finite measure and countable $(\mathcal{H}^j, j)$-rectifiability.

Fractals and quasiconformal mappings

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We review some of the connections between fractals and quasiconformal mappings. This includes a discussion on carpets.

Optimal quantization and Hausdorff measure for Cantor distributions

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The difficulty of finding the best approximation of a probability distribution by a discrete probability with a fixed number of supporting points is known as quantization problem. The theory has its origins in electrical engineering and was developed since the late 1940's mainly for one- and later also for higher-dimensional absolutely continuous finite-dimensional distributions. Because the problem is difficult to solve, one is interested in high-rate asymptotics, i.e. the quantitative behaviour of the approximation error by an increasing number of supporting points. To this end, a first and second order characteristic has been developed, the quantization dimension and the quantization coefficient. As a main result in quantization theory, for probabilities that are absolutely continous with respect to the d-dimensional Lebesgue measure and satisfying a certain moment condition, the quantization dimension exists and equals d. Moreover the quantization coefficient exists as a finite and positive value. In case of singular (continuous) probabilities the situation is different. Every of the following cases can happen:

(i) the quantization dimension does not exist,

(ii) the quantization dimension exists, but not the quantization coefficient,

(iii) the quantization dimension and the quantization coefficient exists.

For the special class of one-dimensional Cantor distributions, which are not necessarily self-similar, several authors achieved results concerning the quantization problem resp. the high-rate asymptotics. Moreover it was shown, that all of the three above mentioned cases can happen for these probabilities. In this talk we focus on a special sub-class of dyadic homogeneous one-dimensional Cantor distributions, which are representatives of case (ii). We will combine the results concerning quantization of these probabilities with a result about the explicit value of the Hausdorff measure for one-dimensional Cantor sets. By doing this, we can express the limes inferior of the sequence, which characterizes the second order quantization asymptotics, in terms of the Hausdorff measure of the supporting Cantor set. Not the same, but a similar linkage exists between the limes superior and the Packing measure. A sketch of the proof will be given.

Boundary Harnack inequality for stable processes on the Sierpinski gasket

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In last few years stable and stable-like processes on fractals attracted considerable attention. During the talk I would like to present the results of my joint paper with Kamil Kaleta. We study the nonnegative harmonic functions of a α-stable jump process on the Sierpiński gasket for $\alpha < 1$. It is shown that a uniform boundary Harnack inequality holds in that case.

During the talk I would like to mention the two main obstacles in proving such results on general fractal sets: construction of smooth functions and estimating the distribution of the process stopped at the first exit from a ball.

Fractal properties of SLE

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The Schramm-Loewner evolution (SLE) is a one parameter family of conformally invariant curves that arises in analysis of critical phenomena in two-dimensional statistical physics. I will give an introduction to these processes with an emphasis on how to derive fractal properties (such as Hausdorff dimension) of the curves.

Localisable moving average processes

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Joint work with K.J. Falconer and J. Levy-Vehel.

We present a particular class of moving average processes which possess a property called "localisability". This means that, at any given point, they admit a "tangent process", in a suitable sense. We give general conditions on the kernel g defining the moving average which ensures that the process is localisable and we characterize the nature of the associated tangent processes. We also present a simulation method for stable moving average. Examples include the reverse Ornstein-Uhlenbeck process and the multistable reverse Ornstein-Uhlenbeck process. In the latter case, the tangent process is, at each time t, a Lévy stable motion with stability index possibly varying with t.

Dimension of some non-normal continued fraction sets

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We consider certain sets of non-normal continued fractions for which the asymptotic frequencies of digit strings oscillate in one or other ways. the Hausdorff dinensions of these sets are shown to be the same value 1/2 as long as they are non-empty. An interesting example among them is the set of "extremely non-normal continued fractions which was previously" conjectured to be of Hausdorff dimension 0.

The Furstenburg problem and the exceptional set of projections

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Given $0 < \alpha \leq 1$, suppose that E is a compact set in the plane such that in every direction there exists a line which intersects E in a set of dimension at least α. How small can be the dimension of E? This is an open question of Furstenburg. I will relate this problem to the problem of exceptional sets of projections, overview known results and make some conjectures.

Classification of finite type fractals

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A fractal of finite type is a self-similar set given by contractions $\{f_i\}_{i=1}^m$ where each piece has only finitely many possible neighbors (up to size). These neighbors can be described by the so called neighbor maps $f_u^{-1}f_v$ with u and v words over the alphabet $\{1,\ldots,m\}$.

For symmetric fractals it is possible to reduce the number of maps we have to take into consideration. The less neighbor maps (up to symmetry) we have, the less complexity we have in the local structure of the fractal. This is the reason we use the number of such maps to classify finite type fractals.

It is remarkable that the most simple fractals in this classification seems to be rather tractable for methods from fractals analysis.

The Geometry of Self-affine Fractals

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We focus on dimension theory. We look at the following three aspects of self-affine attractors:

(1) Dimension formula for the self-affine sets generated by upper triangular matrices;

(2) The dimension of exceptional sets;

(3) The Hausdorff dimension of a random self-affine set.

Random maps and their scaling limits

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A map is a combinatorial, cellular embedding of a graph into a surface. Random maps are a very natural way to endow these surfaces with a random metric. When properly rescaled, these metric spaces converge to random limiting metric spaces, in a way reminiscent of the fact that Brownian paths are limits of discrete random walks. We will review the recent progress made on this subject, with a particular stress on its fractal aspects.

Estimate of the Hausdorff measure of the Sierpinski triangle

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It is well-known that the Hausdorff dimension of the Sierpinski triangle Λ is $s = \log 3/\log 2$. However, it is a long standing open problem to compute the s-dimensional Hausdorff measure of Λ denoted by $\mathcal{H}^s(\Lambda)$. In the literature the best existing estimate is $0.670432 \le \mathcal{H}^s(\Lambda) \le 0.81794$. Using some sofisticated techniques and a computer program we improve the lower bound to 0.77.

An axiomatic approach to fractal dimension

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The axiom system presented in this talk is a modified version of the conditions given by K.J. Falconer in his book "Fractal Geometry". This axiom system is an effect of discussion with Irmina Herburt and Dorette Pronk; our joint paper is to appear in the volume "Convex and Fractal Geometry", Banach Center Publications.

A general result on absolute continuity of non uniform self-similar measures on the real line

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We present a general result on absolute continuity of biased non-uniform self-similar measure on the real line, given by n different contraction and translation rates. The result holds generically in the sense of Lebesgue measure on a certain part of the parameter domain.

About pointwise smoothness of some nondifferentiable functions

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We study the regularity of some historical nondifferentiable function from the Hölderian point of view. A simple method allows to show that most of these functions are monofractal, i.e. that the associated Hölder exponent only takes one finite value.

A multifractal spectrum for the free energy of polymers on disordered trees

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Joint work with Peter Mörters.

We consider a model of directed polymers on a regular tree interacting with a disorder given by independent, identically distributed weights attached to the vertices. Under certain conditions on the distribution of the weights, we observe a phase transition regarding the localization of the polymers. The phase transition can be understood in terms of a multifractal spectrum for the size of the minimal subtree supporting the free energy. At temperatures above the critical value, the spectrum is positive, whereas at criticality the spectrum becomes zero. In the latter case, we show that a single polymer suffices to support the free energy.

On weak convergence to Wick exponential of the fractional Brownian motion

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The Wick-Itô integral with respect to fractional Brownian motion (fBM) is based on Wick product instead of ordinary multiplication. By Sottinen there exists a Donsker-type approximation of the fBM by disturbed random walks. We apply the discrete Wick calculus on these discrete random variables. Thus we show weak convergence to the Wick powers of the fBM. This leads to an approximation of the Wick exponential via Wick products of the fBM.

Geometry of Self-similar sets I: Canonical tilings of the convex hull

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Joint work with Steffen Winter.

An iterated function system Φ consisting of contractive affine mappings has a unique attractor $F \subseteq \mathbb{R}^d$ which is invariant under the action of the system, as was shown by Hutchinson. I will describe how the action of the function system naturally produces a tiling $\mathcal{T}$ of the convex hull of the attractor. These tiles form a collection of sets whose geometry is typically much simpler than that of F, yet retains key information about both F and Φ. In particular, the tiles encode all the scaling data of Φ. There are two primary motivations for this construction. On the theoretical

side, the tiling $\mathcal{T}$ is the foundation for the higher-dimensional extension of the theory of *complex dimensions* which was developed by Lapidus and van Frankenhuijsen for the 1-dimensional case. On the practical side, the tiling provides a relatively painless way to obtain a tube formula (that is, an explicit function of epsilon that gives the volume of the region which lies within epsilon of F) for self-similar sets. The tube formula is obtained by combining classical results of convex geometry due to Steiner with the theory of complex dimensions.

Multifractional stochastic volatility models

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Joint work with Ântoine Ayache.

Hull and White and other authors in the field of Mathematical Finance have introduced the so-called stochastic volatility models for taking into account some randomness which is specific to volatility (this randomness is due to exogenous arrivals of information). Later, Comte and Renault have proposed to replace the Brownian dynamic in these models by a Fractional Brownian Motion in order to make them more realistic. More recently, by using the notion of Generalized Quadratic Variations, Gloter and Hoffmann have constructed estimators of the parameters of Fractional stochastic volatility models. The goal of our talk is to introduce the Multifractional stochastic volatility models and to extend some results of Gloter and Hoffmann to these new models; the non-stationarity of the increments makes the proofs more difficult than that in the fractional case. At last, let us mention that the Multifractional stochastic volatility models allow to take into account the fact that the local regularity of financial signals changes from one place to another.

Probabilistic characterisation of Besov-Lipschitz spaces on fractals and measure metric spaces

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Suppose that (X, ρ, μ) is a measure metric space, with μ being a $d-$regular measure (e.g. a nested fractal, or the Sierpiński carpet, or a p.c.f. self-similar set). Further, suppose that X supports a 'fractional diffusion' – i.e. a diffusion whose transition density with respect to μ satisfies certain two-sided exponential estimate.

It is known (Jonsson, Pietruska-Paluba, Grigoryan-Hu-Lau, and others) that the domain of the Dirichlet form related to this diffusion is a Besov-Lipschitz space (see Jonsson '96) $Lip(\frac{d_w}{2}, 2, \infty)(X)$, d_w being the walk dimension of X.

The Besov-Lipschitz $Lip(\alpha, p, q)(X)$ spaces were introduced for a wide range of parameters: $\alpha > 0, 1 \leq p, q \leq \infty$, and the known probabilistic interpretation was known only for $\alpha = \frac{d_w}{2}$, $p = 2$, $q = \infty$.

In this talk, we will present the probabilistic interpretation of the spaces $Lip(\alpha, p, q)(X)$, with general parameters α, p, q.

Large porosity and dimension of sets in metric spaces

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We investigate how the geometry of a metric space is related to the dimension of its lower-porous subsets. The asymtotical behaviour of dimension when porosity tends to zero is well understood. To obtain the estimates in this case it is enough to assume that the metric space is doubling.

In this talk we focus on the case where porosity is close to its maximum. It is easy to notice that if one wants to obtain the same kind of asymtotical dimension result as in the Euclidean case one has to impose heavy restrictions on the metric space. For example assuming that the space is Ahlfors regular geodesic metric space is not enough.

By comparing the metric spaces to the Euclidean ones we are able to write down conditions for the metric spaces that guarantee the desired behaviour of their porous subsets. These conditions are satisfied, for example, by normed vector spaces and Heisenberg groups.

Geometrical realizations of hyperbolic substitution dynamical systems

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Symbolic dynamical systems play an important role in the study of dynamical systems, and substitution dynamical systems is a very important class among symbolic dynamical systems.

Long time ago, the problem of understanding the spectra of substitution dynamical systems has been raised [5]. In particular, it is conjectured that substitution dynamical systems of Pisot type have purely discrete spectrum. This conjecture remains open.

To study the geometrical realization of substitution dynamical system is one way to deal with the spectral problem, which is initialed by G. Rauzy. The realization is usually called *atomic surfaces*, or *Rauzy fractal* [6, 1].

For *unimodular Pisot substitution*, the atomic surfaces forms a self-similar tiling system, which is an analogue of the famous Penrose tiling [4].

Also, the geometrical realization can be applied to the study of β-numeration system, provided that β is a Pisot number [3].

For Pisot substitution, the spectral properties, the tiling properties of the atomic surfaces and the number theoretical properties of the β-numeration systems are all governed by a *super-coincidence property*. It is conjectured that all Pisot substitutions possess of the super-coincidence property [3].

If a hyperbolic substitution is not of Pisot type, then the atomic surfaces is a fractal [2].

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[2] M. Furukado, S. Ito and H. Rao, Atomic surfaces of hyperbolic unimodular substitutions. Preprint 2007.

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[4] S. Ito and H. Rao, Atomic surfaces, tilings and coincidences I. Irreducible case. *Israel J. Math.* **153** (2006), 129-156.

[5] M. Queffelec, Substitution Dynamical Systems - Spectral Analysis. Lecture Notes in Math. **1294** (1987), Springer-Verlag, New York.

[6] G. Rauzy, Nombres algébriques et sustitutions. *Bull. Soc. Math. France* **110** (1982), no. 2, 147-178.

Approximation of (fractal) sets by parallel neighbourhoods

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In the first part, some geometric properties of the parallel r-neighbourhoods $A_r = \{x : \mathrm{dist}(x, A) \leq r\}$ $(r > 0)$ of a set $A \subseteq \mathbb{R}^d$ will be given, including the volume, surface area and curvature measures. The question of stability under approximations will be addressed as well. After then, an application when A is the path of a Brownian motion will be presented. In particular, formulae for the mean volume and surface area of the parallel neighbourhood of a trajectory running until a given time T will be given, and certain particular results for curvature measures will be mentioned.

Extended method of indivisibles and non-Euclidean geometric measure representations

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According to the method of indivisibles of Cavalieri and Torricelli, one can say that two solids A and B have equal volumes if they are situated between the same two fixed spheres, centered at the same point P, and if for any sphere S in between and also being centered at P the intersections of A and B with S have the same surface contents.
It follows from the co-area formula of measure theory that this principle works for $l_{n,p}$-spheres if and only if p equals 2.
It has been recently shown in [1], however, that an analogous principle holds for $l_{n,p}$-spheres if the notion of the Euclidean surface content is replaced by that of the $l_{n,p}$-generalized surface content.

It will be demonstrated in this talk that the $l_{n,p}$-generalized surface measure is based upon a suitably defined non-Euclidean Minkowski geometry if $p \geq 1$ and upon the geometry which is generated by the Minkowsky functional of a suitably defined star-shaped set if $0 < p < 1$. Making use of this notion, a general geometric measure representation formula for continuous $l_{n,p}$-symmetric distributions will be derived. These distributions extend the class of p-generalized normal laws and allow to model heavy or light tails.
First applications deal with generalizations of classical statistical distributions.
The present approach is closely connected in the two-dimensional case with solving the $l_{2,p}$-isoperimetric problem and results in a natural generalization of the circle number π, see [2].

[1] Richter, W.-D., Generalized spherical and simplicial coordinates. J. Math. Anal. Appl. 336(2007), 1187-1202; doi.10.1016/j.jmaa.2007.03.047.
[2] Richter, W.-D., On $l_{2,p}$-circle numbers. Lith. Math. J., 48(2008),2, 228-234.

Shot-noise cascades

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Scaling and related phenomena form a striking and crucial component directly impact performance in a variety of applications, notably in networking and finance. We study the *Cascade of Pulses* $\{Q_r(t)\}_t$

$$Q_r(t) := \prod_{R_i > r} \pi\left(W_i, \tfrac{t-T_i}{R_i}\right). \tag{1}$$

where $(T_i, R_i; W_i)$ form a marked Poisson Point Process on $\mathbb{R} \times (0,1]$ with density $dm(t,a)$, a setting which allows for infinitely divisible scaling, strict stationarity and for deviations from powerlaws. Examples of interest include

$$\pi(w,t) := \begin{cases} 1 + (w-1)h(t) & \text{multiplicative shot-noise pulse,} \\ w^{k(t)} & \text{exponential shot-noise pulse.} \end{cases} \tag{2}$$

which reduce to the well-known cylindrical product of pulses when h or k is the indicator of $[-1/2, 1/2[$; also, the exponential shot-noise cascade can be written as an infinitely divisible cascade over a Poisson Counting Measure.

To the best of our knowledge, this is the first analysis of the cascade with pulses that are *not compactly supported.* The pulses having potentially infinite support precludes the use of certain standard techniques for establishing its multifractal properties. As a first step towards overcoming this hurdle, we establish moment and regularity conditions under which an appropriate limiting cascade $A(t) = \lim_{r\to 0} \int_0^t \frac{Q_r(s)}{\mathbb{E}[Q_r(s)]} ds$ can be defined and exhibits the scaling

$$\mathbb{E}|A(t+\delta) - A(t)|^q \simeq \delta^q \cdot \frac{\mathbb{E}[Q_r(t)^q]}{\mathbb{E}[Q_r(t)]^q} = \delta^{q-c(\rho(q)-q\rho(1))}$$

where $\rho(q) = \mathbb{E}[\int_{-\infty}^{\infty} \pi(W,u)^q - 1du]$ and $dm(t,a) = c/a2dtda$. Conditions and formulas take simple forms in the shot-noise cases. Financial support in part by the Fernfachhochschule Schweiz and by NSF grant ANI-0338856.

Dimension of some self-similar measures with overlaps

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We study the dimension of some self-similar measures with overlaps, those obtained from systems of homotheties with centres in a lattice where the contraction ratios are all equal to the inverse of a natural number L. We obtain the local dimension of the measure as the Shannon entropy rate of an associated hidden Markov chain divided by the logarithm of L. This result is useful in the study of the absolute continuity or singularity of the measure and provides two sequences convergent to the dimension of the measure, one of them non-increasing and the other non-decreasing, which allows us to obtain estimates of the dimension.

Aspects of Fractals in Non-Commutative Geometry

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Non–commutative geometry is discussed in many contexts. The common idea is that one deals with a function algebra over the space that one is interested in and the description of its geometry is made free from the concept of a point, interval, disc... . We will discuss the definition of a spectral triple (a non–commutative space) and give an examples of spectral triples on the circle and a self similar set in the unit interval. In the talk we aim to introduce the idea of a non–commutative integral and metric and give results showing that these do in fact coincide with the usual notion of integration and distance. Finally, time permitting I hope to show how one may introduce the notion of mulifractal analysis into the language of spectral triples and give some results and conjectures on this.

A multifractal mass transfence principle and the structure of a typical sequence

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Let μ be a Gibbs measure of the doubling map T of the circle. A (deterministic) mass transference principle is obtained for such a Gibbs measure which is in general multifractal. More precisely, for a given sequence of decreasing intervals the knowledge of the Hausdorff-Billinglsey dimension of the limsup-set gives you the information of the Hausdorff dimension of the limsup-set with scaled radii. Such a principle was shown by Beresnevich and Velani [1] for mono-fractal measures. For

a given deterministic sequence it is in general hard to check the assumption of the principle.

However, we will show that a typical sequence for the measure fulfills the assumptions of the mass transference principle. In the symbolic language we completely describe the combinatorial structure of a typical relatively short sequence, In particular we can describe the occurrence of "atypical" relatively long words. Our results have a direct and deep number-theoretical interpretation via inhomogeneous dyadic Diophantine approximation by numbers belonging to a typical orbit.

[1] Beresnevich, Victor; Velani, Sanju A mass transference principle and the Duffin-Schaeffer conjecture for Hausdorff measures. Ann. of Math. (2) 164 (2006), no. 3, 971–992.

A Markov process with random singularity spectrum

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In this work we construct a Markov process with a random singularity spectrum. The dependence of the singularity spectrum on the sample paths is given explicitely. The multifractal analysis of this process is based on new "ubiquity theorems". These theorems have important consequences in Diophantine approximation.

Multifractal products of stationary diffusion processes

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Joint work with V. Anh (Brisbane) and N. Leonenko (Cardiff).

We investigate the multifractal moment-scalings for products of an exponential process determined by the stationary diffusion which is defined as the weak solution of mean-reverting SDE

$$dX(t) = -\theta(X(t) - \mu)dt + \sqrt{v(X(t))}dB(t),\ t \geq 0, \tag{3}$$

where $\theta > 0, \mu \in (l, r),\ -\infty \leq l < r \leq \infty$, v is a non-negative function on the interval (l, r), and $\{B(t), t \geq 0\}$ is the standard Brownian motion.

The work shows that Kahane's T-martingale scheme in the form of P. Mannersalo, I. Norris and R. Riedi (AAP 2002) can be worked out for the exponential process $Y(t) := e^{X(t)-c}$(c is a normalizing constant). We provide three illustrative examples of normal, gamma and beta distributions. The paper will appear in *Stochastic Analysis and Applications*.

Smooth and affine rigidity of self-similar sets

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Given a self-similar set E, what are all the affine functions f mapping E into itself? In two recent papers by D.J. Feng and Y. Wang and by M. Elekes, T. Keleti and A. Máthé, it has been shown that in many cases, such as for the ternary Cantor set, the family of such functions is rigid (in a sense I will explain) and can be described explicitly. I will show how some of these results can be recovered and extended as corollaries of theorems on sums of Cantor sets by C. Moreira and by Y. Peres and myself. As an example, I will show that there are no non-linear analytic functions mapping the ternary Cantor set into itself.

Algebraic difference of random Cantor sets I

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The study of the algebraic difference

$$F_2 - F_1 = \{y - x : x \in F_1, y \in F_2\}$$

of two dynamically defined Cantor sets $F_1, F_2 \subset \mathbb{R}$ was motivated by the reserch of Palis and Takens in regards with the unfolding of homoclinic tangency in some one-parameter families of surface diffeomorphisms.

Palis conjectured that if

$$\dim_{\mathrm{H}} F_1 + \dim_{\mathrm{H}} F_2 > 1$$

then *generically* it should be true that

$$F_2 - F_1 \text{ contains an interval.}$$

For generic dynamically generated *non-linear* Cantor sets this was proved in 2001 by de Moreira and Yoccoz. The problem is open for generic linear Cantor sets. In this talk I will speak about related results for random Cantor sets.

Estimation of ordinal pattern probabilities in fractional Brownian motion

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Joint work with Karsten Keller.

Ordinal patterns describe the ordering of successive equidistant values of a time series in terms of permutations. Based on the distribution of ordinal patterns one can define statistics to quantify the complexity of a time series and the underlying model, respectively. If the underlying model is a dynamical system, one particularly interesting statistic is the permutation entropy introduced by Bandt and Pompe [2], which is the Shannon entropy of ordinal pattern distributions. Bandt, Keller and Pompe [1] have shown that for piecewise monotonically linear interval maps the permutation entropy coincides with the Kolmogorov-Smirnov entropy.

A much simpler statistic based on (empirical) ordinal pattern distributions is the frequency of changes between monotonic upward and downward in a time series. Bandt and Shiha [3] have shown that for equidistant discretizations of fractional Brownian motion (fBm), the higher the Hurst parameter the less the probability of a change. The estimator of the Hurst parameter obtained by plugging the frequency of changes in a realization into this monotonic relation has been known for some time as 'Zero Crossings' (ZC) estimator [4].

We discuss general properties of estimators of ordinal pattern probabilities in equidistant discretizations of fBm [5]. As it turns out, the distribution of ordinal patterns is invariant with respect to the chosen sampling rate. Further, according to the sufficiency principle, it is 'better' not to simply estimate the probability of an ordinal pattern by its sample frequency, but to take as an estimate the average of the sample frequencies of the pattern itself and its 'time' and 'spatial' reversals. The estimators obtained this way are strongly consistent, and asymptotically normal if the Hurst parameter is less than 3/4.

We derive confidence intervals for the ZC estimates of the Hurst parameter. Simulation studies show that the confidence intervals are reliable also for small sample sizes and values of the true (unknown) Hurst parameter larger than 3/4. In comparison to well-known semi-parametric estimators, the ZC estimator has slightly less variance and remarkably small bias.

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[4] Coeurjolly, J. F., Simulation and identification of the fractional Brownian motion: A bibliographical and comparative study. *J. Stat. Software* 5 (2000).

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Multifractal spectra of in-homogeneous self-similar measures: results and applications

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Joint work with L. Olsen.

First we will discuss the multifractal spectra of in-homogeneous self-similar measures satisfying the In-homogeneous Open Set Condition. Then we will present several applications of our results. Many of them are related to the notoriously difficult problem of computing (or simply obtaining non-trivial bounds) for the multifractal spectra of of self-similar measures not satisfying the Open Set Condition. More precisely, we will show that our results provide a systematic approach to obtain non-trivial bounds (and in some cases even exact values) for the multifractal spectra of several large and interesting classes of self-similar measures not satisfying the Open Set Condition, including, for example, self-similar measures supported on the $(0, 1, 3)$-set of γ-expansions with deleted digits.

The discontinuity set of a function as the hyperbolic boundary of a graph

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Given a function of bounded variation on the unit interval, we present a construction under which the set of non-differentiable points is in a one-to-one relation with the boundary of an infinite, tree-like graph, which has been introduced in [1], and is called 'augmented tree'. By showing hyperbolicity of this graph, we can use recent results [1,2,3] about the Hausdorff dimension of the hyperbolic boundary in terms of drift and asymptotic entropy of a random walk on the augmented tree. The construction is extended to functions with bounded quadratic variation and any property which - if present on an interval - may inherit under restriction of the function to a subinterval. Discussed examples of functions and 'hereditary properties' include the Cantor function and absolute continuity, the graph of the Brownian bridge together with Hoelder continuity, and other random functions with an ergodic, scale- and shift-invariant law, together with jump-like discontinuities.

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[2] V. Kaimanovich: 'Hausdorff dimension of the harmonic measure on trees', Ergod. Th. Dyn. Sys. (1998), 631-660

[3] Le Prince: 'A relation between dimension of the harmonic measure, entropy and drift for a random walk on a hyperbolic space', Elect. Comm. in Probab. 13(2008), 45-53

Quasisymmetric conjugacy between rational maps and iterated function systems

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Joint work with Ilgar Eroglu and Steffen Rohde.

We consider linear iterated function systems (IFS) with a constant contraction ratio in the plane for which the "overlap set" is finite, and which are "invertible" on the attractor A, in the sense that there is a continuous surjection q on A whose inverse branches are the contractions

of the IFS. We suppose also that there is a rational function p with the Julia set J such that (A, q) and (J, p) are conjugate. We prove that if A has bounded turning and p has no parabolic cycles, then the conjugacy is quasisymmetric. This result is applied to some specific examples, including an uncountable family of quadratic maps, the Sierpinski gasket, and the hexagasket.

Stochastic Homogenization on Self-similar Fractals

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We will use the random selfsimilar structure to generalize the concept of stochastic two scale convergence extending some ideas elaborated in the deterministic case by Kolumban [3].

Let $L^2(\Omega \times O, B(\Omega) \times C_b(\mathbb{R}^n))$: measurable random functions $u : \Omega \times O \to B(\Omega) \times C_b(\mathbb{R}^n)$ such that $\| u \| \in L^2(\Omega \times O)$.

If $\Phi \in L^2(\Omega \times O, B(\Omega) \times C_b(\mathbb{R}^n))$ then the following convergence result holds:

$$\lim_{k\to\infty} \sum_{|\sigma|=k} \int_\Omega r_\sigma^s \int_X \Phi(\varphi_\sigma(z), z) d\nu(z) d\gamma = \int_\Omega \int_{K^*} \left[\int_X \Phi(x, y) d\nu(y) \right] d\mu^*(x) d\gamma,$$

where K^* and μ^* are the invariant set and the invariant measure, respectively of the RIFS $\{\varphi_1, ..., \varphi_m\}$.

[1] **G. Allaire:** *Homogenization and two-scale convergence*, SIAM J. Math. Anal. 23(6)(1992), 1482-1518.

[2] **J.E.Hutchinson, L.Rüschendorf:** *Selfsimilar Fractals and Selfsimilar Random Fractals,* Progress in Probability, 46, (2000), 109-123.

[3] **J. Kolumbán:** *Two scale convergence of measures* (to appear).

Dynamical Percolation

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The dynamical percolation model will be introduced and some of the earlier results for it will be mentioned. Then, we will give a brief outline of the proof, obtained jointly with Oded Schramm, that in 2 dimensions there are exceptional times. This will be connected to 'noise sensitivity' and the Fourier spectrum for percolation. I will also briefly describe a recent theorem by Garban, Pete and Schramm which describes very specifically the behavior of the Fourier spectrum for percolation.

Upper porosity of packing type measures

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We discuss some porosity properties of measures on metric spaces. The main result gives sufficient conditions for packing type measures to be upper porous. Intrestingly, similar results are not valid for Hausdorff measures.

A class of random fractals and their geometric properties

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Structures arising in earth siences are often modelled as random tessellations (mosaics) as considered in stochastic geometry. For describing cracks one uses the notion of iteration of tessellations. It is a disadvantage of the known models that they are not able to describe very fine structures such as hair line cracks. We modified the notion of iteration and considered limit sets of randomly iterated tessellations. Expecting some kind of fractal properties in the limit, we restricted ourself for a first model to so-called self-similar tessellations. Here it is possible to calculate the Hausdorff dimension as well as the exact Hausdorff function for the limit sets. Also fractal curvatures and mean fractal curvatures will be considered. The theoretical results are compared with numerical computations.

Probabilistic approach to transformations preserving the Hausdorff-Besicovitch dimension

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Joint work with Sergio Albeverio, Yuval Peres and Mykola Pratsiovytyi.

F.Klein's work made the group theoretic approach to geometry well known. What is "the fractal geometry" from the group theoretic approach? One can find the words "fractal geometry" in many works, even though no formal definition is given. Monograph [2] contains an attempt to answer the question saying that "... one approach to fractal geometry is to regard two sets as 'the same' if there is a bi-Lipschitz mapping between them". Fractal geometry is in this sense the study of invariants of bi-Lipschitz transformations. A view on fractal geometry in the same sense, but with a more general definition of allowable mappings has been proposed in [1].

There exist transformations (one to one mappings into itself) of a metric space (M, ρ) which preserve the Hausdorff-Besicovitch dimension of every subset $E \subset M$. Such transformations are said to be DP-transformations of (M, ρ). The group G of all DP-transformations is essentially wider than the group of bi-Lipschitz transformations. We view fractal geometry as a mathematical discipline studying invariants of transformations from the DP-group. The group G includes many interesting subgroups. In particular, G contains the group of all affine transformations (and thus, affine geometry may be considered as a part of fractal geometry).

If one admits the above point of view, then the following area are of interest:

1) The description of the group G and an investigation of conditions under which a transformation f is DP.

2) The study other invariants of the group G.

3) The study the subgroups of G.

4) Develop applications of DP-transformations.

Our talk will be devoted to the case of continuous transformations of R^1, which is equivalent to the investigation of DP-properties of probability distribution functions on $[0, 1]$. We show that methods of the multilevel fractal analysis of singularly continuous probability measures are most appropriate ones for this purpose.

We show that the group G is essentially wider than the group of bi-Lipschitz mappings (for instance the sets $N_F^{\infty} = \{x : F^{'}(x) = +\infty\}$ and $N_F^0 = \{x : F^{'}(x) = 0\}$ can be everywhere dense sets of full Hausdorff dimension).

We discuss general properties of DP-transformations as well as DP-transformations generated by special classes of probability distributions. A number of examples and counterexamples will be presented. Applications of DP-transformations will also be discussed.

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[3] Turbin A.F., Pratsiovytyi M.V., Fractal sets functions and distributions, Naukova Dumka, Kiev, 1992

Systems of two iterated functions over the skew field of quaternions

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Iterated linear function systems $f_0(z) = qz + a$, $f_1(z) = qz + b$ over the field of complex numbers have been investigated since 1985 (Barnsley and Harrington). The question of connectedness of their attractors is widely studied. Present paper introduces iterated function systems $f_0(z) = qzp + a$, $f_1(z) = qzp + b$ over the skew field of quaternions. We suggest a simplification of a form of such systems and consider the structure of their attractors.

Iterated function systems over the field of complex numbers $\mathbb{C}$ of the form

$$\begin{cases} f_0(z) = qz + a, \\ f_1(z) = qz + b, \qquad q, z, a, b \in \mathbb{C},\ |q| < 1 \end{cases} \tag{4}$$

were studied before by Barnsley, Harrington, Bandt, Solomyak etc. The problem of connectedness of their attractors was thoroughly investigated in the form of topological properties of Mandelbrot set. For these systems the question of similarity is trivial: given fixed q and varying a, b the attractor of the system (4) is always similar to the attractor of a system

$$\begin{cases} f_0(z) = qz, \\ f_1(z) = qz + 1, \qquad q, z \in \mathbb{C},\ |q| < 1. \end{cases} \tag{5}$$

Thus it's sufficiently to explore the case $a = 0, b = 1$.

We will discuss iterated function systems over the skew field of quaternions $\mathbb{H}$ of the form

$$\begin{cases} f_0(z) = qzp + a, \\ f_1(z) = qzp + b, \qquad q, p, z, a, b \in \mathbb{H},\ |qp| < 1. \end{cases} \tag{6}$$

It turns out that in the given case with fixed q, p and varying a, b attractors are no more similar.

In this work we show to which most simple form the system (6) can be deduced, give an example of four-, three-, two- and one-dimensional attractors of this system.

We also show that in the particular case when $p = \overline{q}$ attractor lies in three-dimensional subspace and is isometrical to one defined by pair of similitude maps in $\mathbb{R}^3$.

In the case $p = 1$ attractor lies in two-dimensional plane and is isometrical to attractor of the system (4) in $\mathbb{C}$.

Geometry of Self-similar sets II: Parallel sets and tube formulas

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Joint work with Erin Pearse.

The ε-parallel sets of a self-similar set F and of its associated canonical tiling $\mathcal{T}$ (see the talk by Erin Pearse) do not always coincide and then the tube formula for $\mathcal{T}$ is not a tube formula for F (i.e. it does not describe the volume of the parallel sets of F). We characterize the situation when the parallel sets coincide such that the tube formula for F can be obtained from $\mathcal{T}$. We also discuss a generalization of the tiling construction which replaces the convex hull of F by an arbitrary open set satisfying the open set condition for F. The construction applies to a larger class of self-similar sets and it is more flexible. For many self-similar sets F, the starting set for the tiling construction can be chosen in such a way that the parallel sets of the tiling and of F coincide, allowing again to use the tiling to obtain a tube formula for F.
